

Math 70 MG 5/e 8.2

To derive the quadratic formula, complete the square using $ax^2 + bx + c = 0$.

$$ax^2 + bx = -c$$

collect variables left and constant right

$$a(x^2 + \frac{b}{a}x) = -c$$

factor out a

$$\# = \frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$$

take half of coefficient of x.

$$\#^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

square result

$$a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) = -c + a \cdot \frac{b^2}{4a^2}$$

add to LHS inside parentheses

distribute and add to RHS.

$$a\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a} - c$$

factor LHS
simplify RHS

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

divide both sides by a.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

write RHS with LCD

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

subtract RHS

$$x + \frac{b}{2a} = \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

solve by square root property

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

simplify RHS

$$x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

isolate x

Voila!
The
Quadratic
Formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

add/subtract fractions with LCD.